Shortest Cycles With Monotone Submodular Costs

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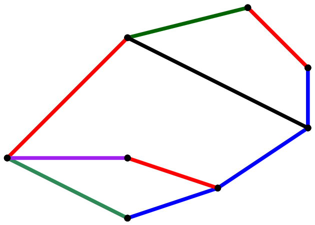
23 January 2023

The Minimum Color Cycle Problem

Minimum Color Cycle

Input: Edge-colored undirected graph.

Output: Cycle with the minimum number of different colors.



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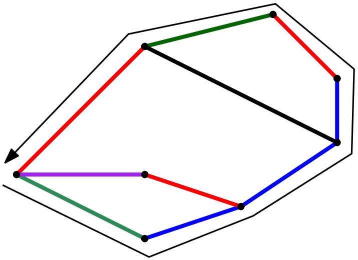

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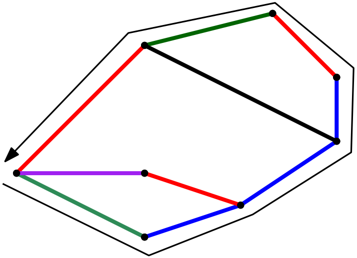

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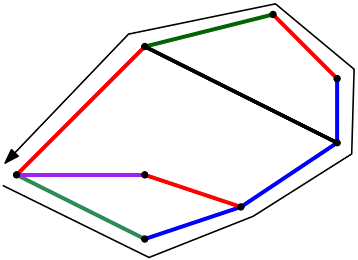

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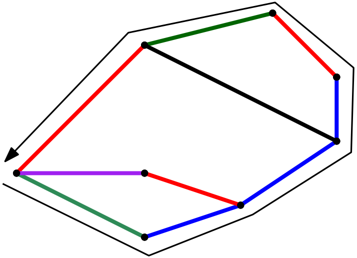
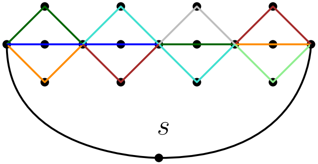


- Polynomial-time or NP-hard?
- If all edges have different colors, equivalent to shortest cycle
- If we require the cycle to include a specified vertex *s*, then NP-hard [Broersma, Li, Woeginger, Zhang '05]



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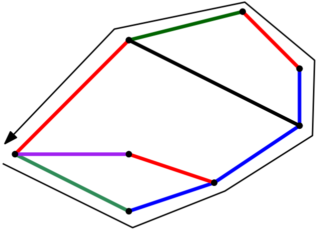

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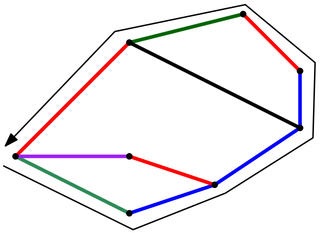

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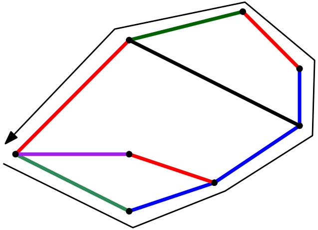

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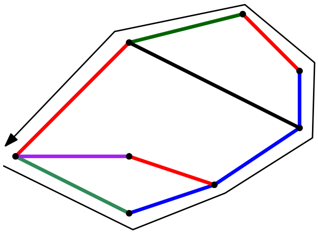

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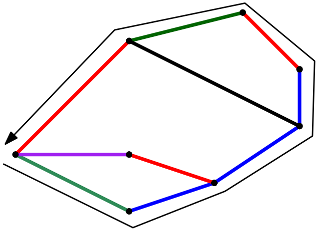

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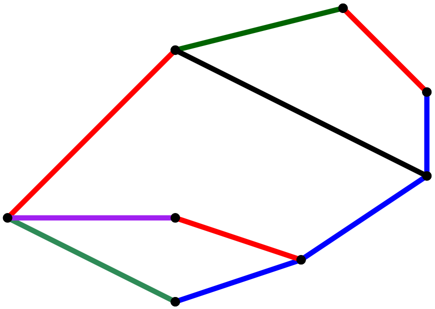
Note: The approximation scheme implies the $n^{\mathcal{O}(\log \mathrm{OPT})}$ algorithm

Algorithm

Goal: Recursive algorithm with $\mathcal{O}(\log \mathrm{OPT})$ recursion levels



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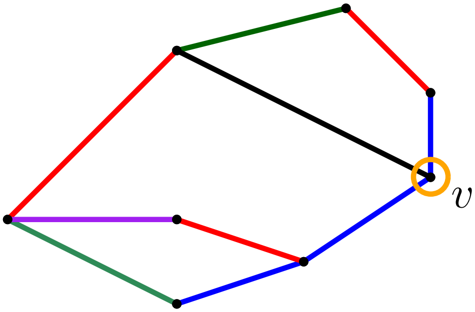

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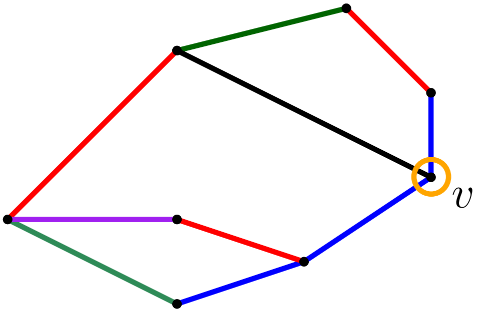

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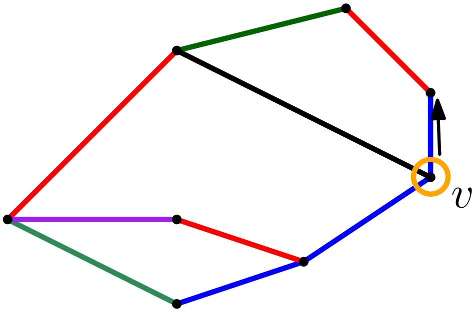

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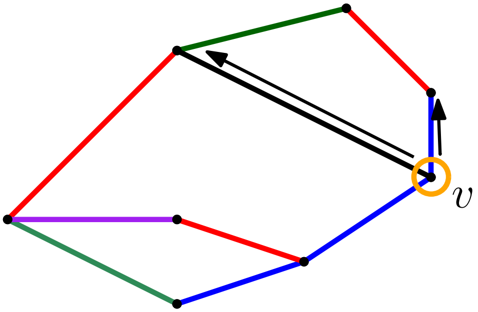

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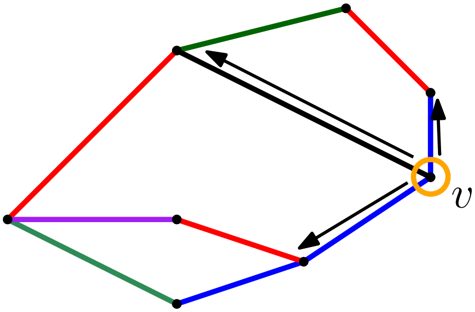

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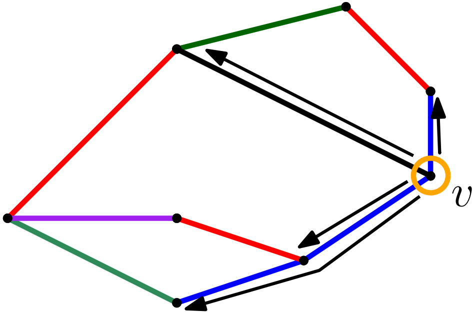

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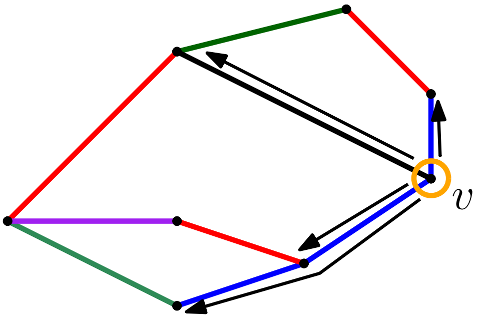

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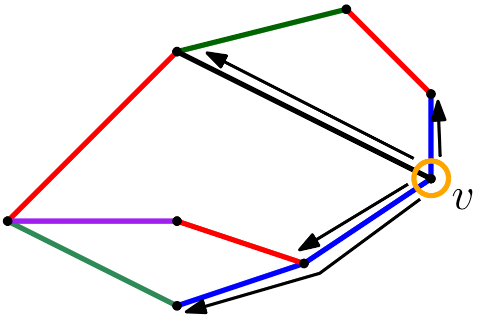

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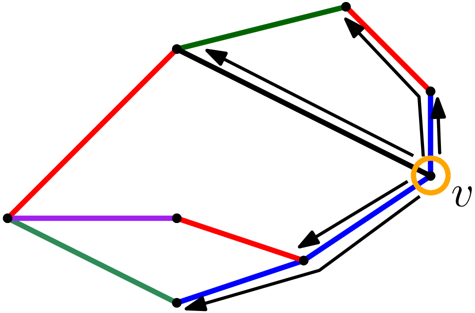

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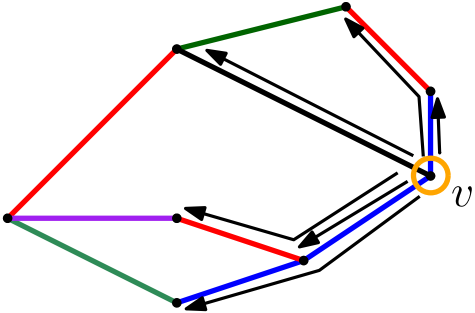

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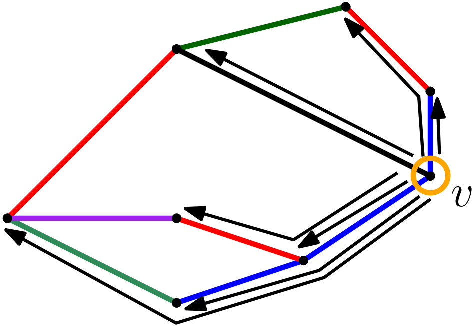

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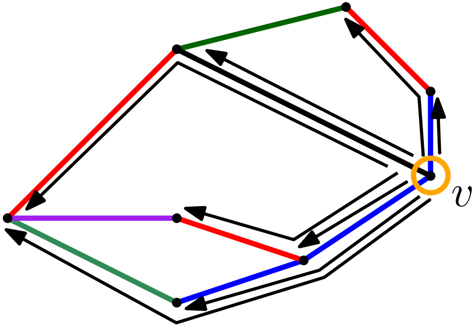

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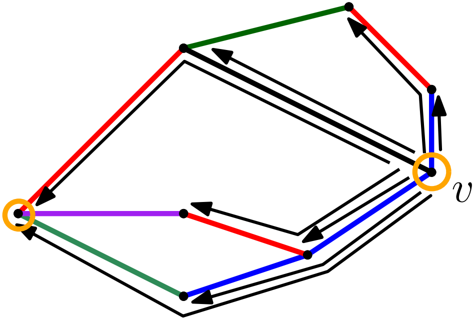

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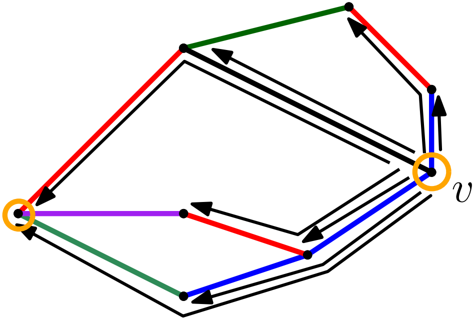

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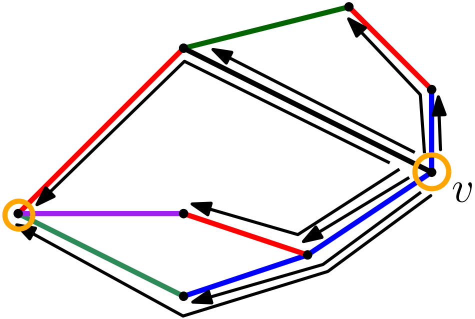

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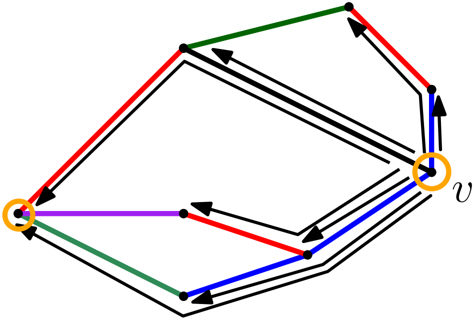

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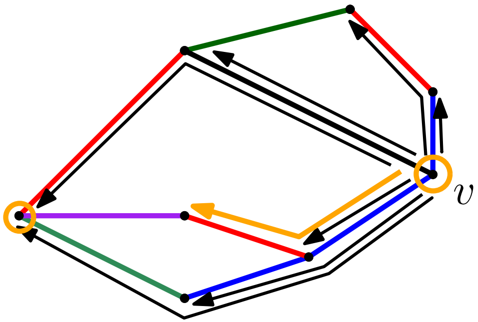

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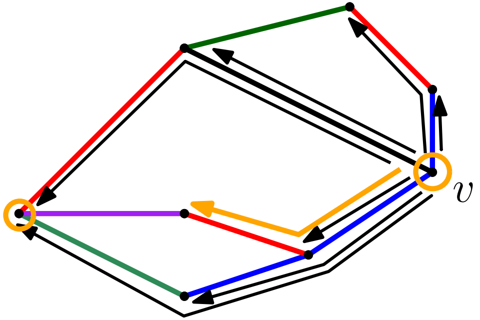

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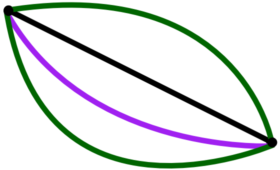

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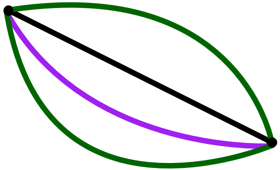

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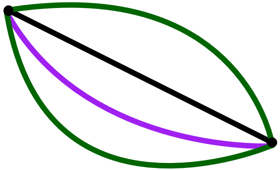

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In contrast:

- Exponential number of queries required to $\mathcal{O}(n^{2/3-\varepsilon})$-approximate minimum submodular cycle through specified vertex [Goel, Karande, Tripathi, Wang '09]

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- Similar lower bounds also for perfect matching, spanning tree, and (s,t)-cut [Goel, Karande, Tripathi, Wang '09; Jegelka and Bilmes '09]

Tightness

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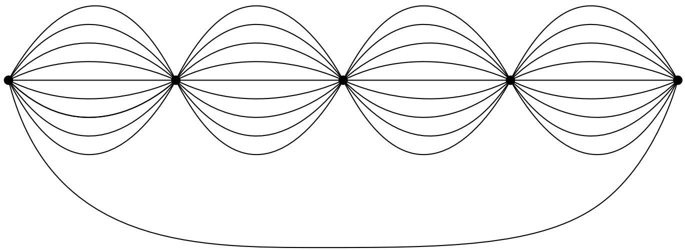
Proof idea:



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For every ε, at least $n^{\log_2(1/\varepsilon)-\mathcal{O}(1)}$ queries are required to $(1+\varepsilon)$-approximate shortest submodular cycle.

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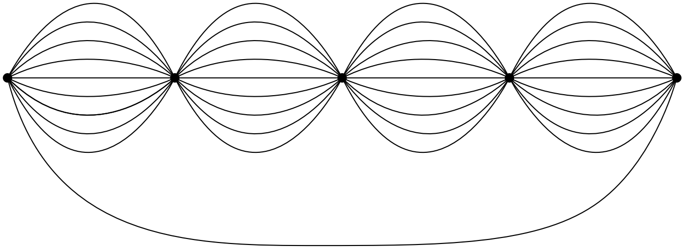

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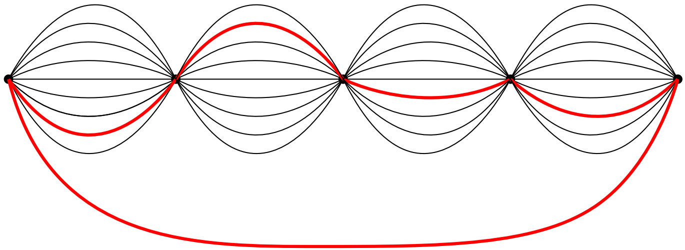

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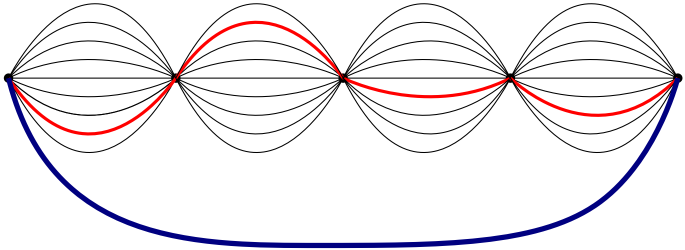

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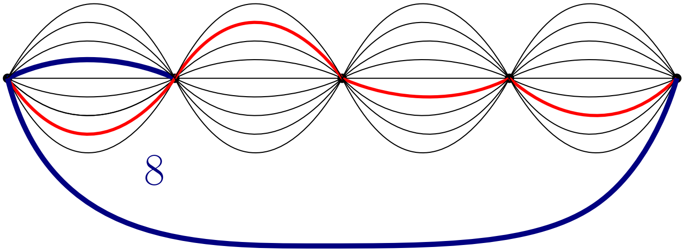

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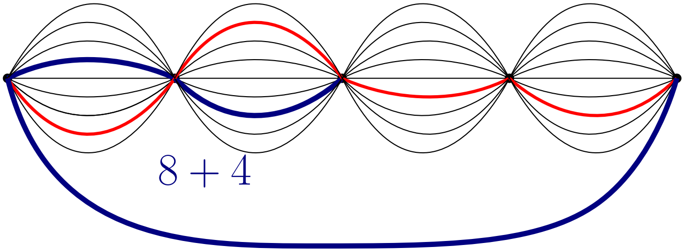

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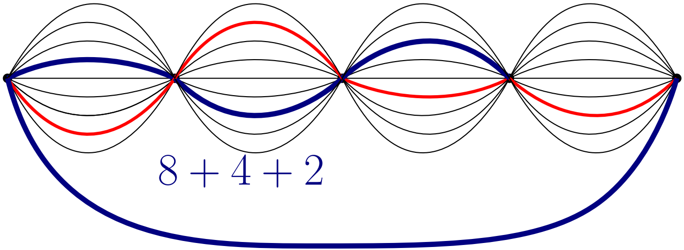

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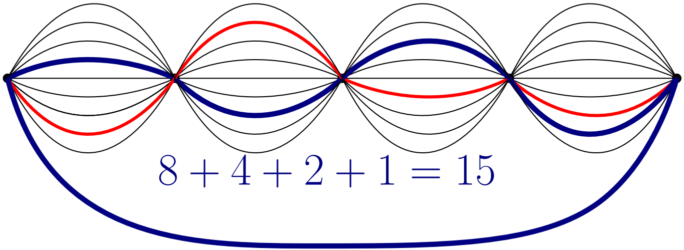

Connection to min-cut

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    - Which was proven to be optimal (for general graphs only!) by [Jaffke, Lima, Masarik, Pilipczuk & Souza; SODA'23]

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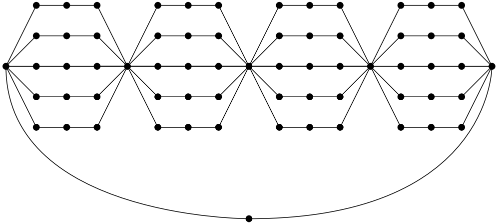

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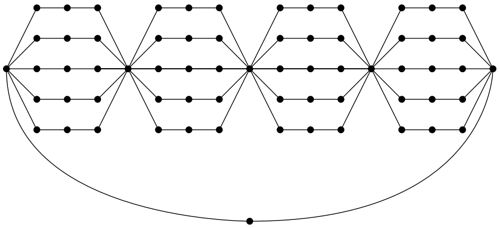

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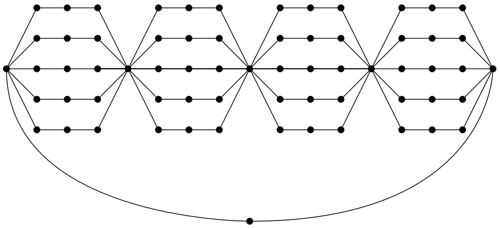

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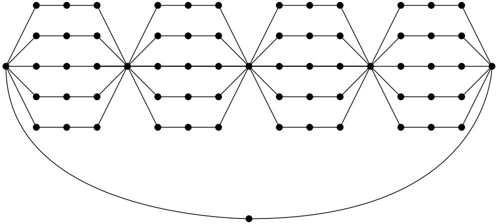

Thank you!