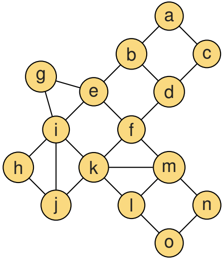
Dynamic Treewidth in Logarithmic Time

Tuukka Korhonen

FOCS 2025

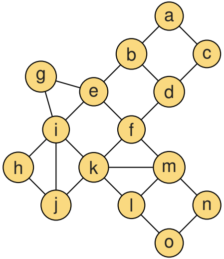
16 December 2025

Treewidth



Graph G

Treewidth



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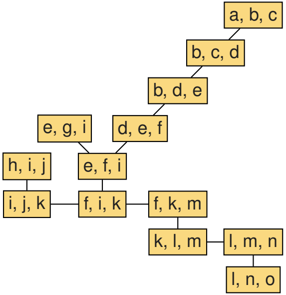

A tree decomposition of *G*

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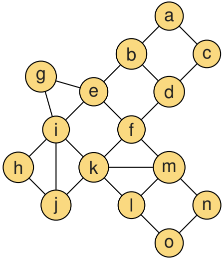

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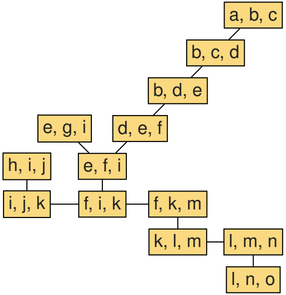

A tree decomposition of G

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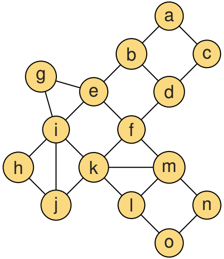

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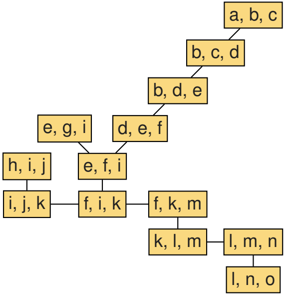

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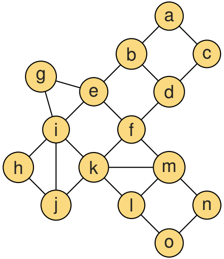

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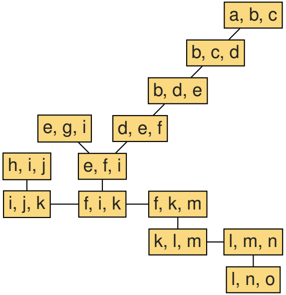

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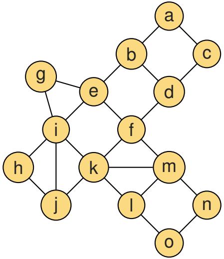

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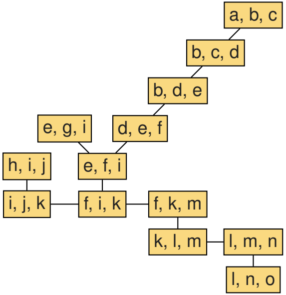

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4. Width = maximum bag size -1

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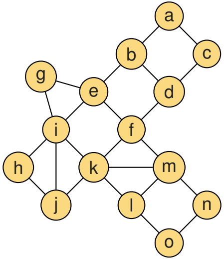

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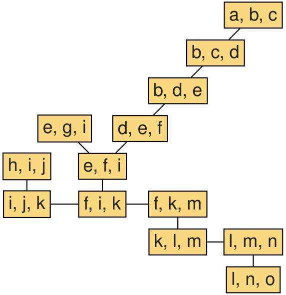

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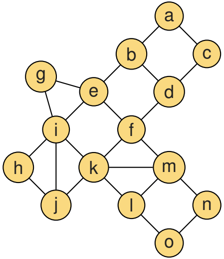

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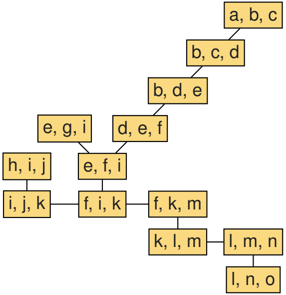

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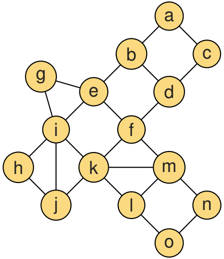

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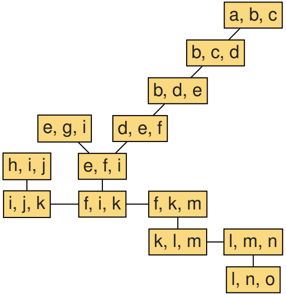

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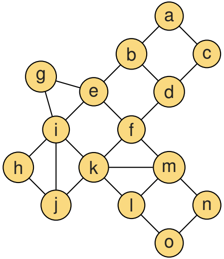

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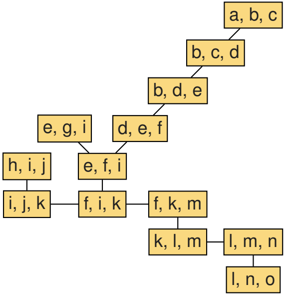

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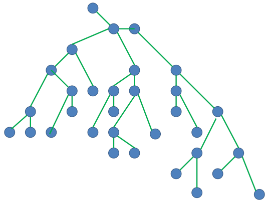
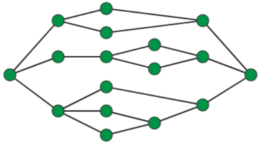
[Robertson & Seymour'84, Arnborg & Proskurowski'89, Bertele & Brioschi'72, Halin'76]

Treewidth of graphs

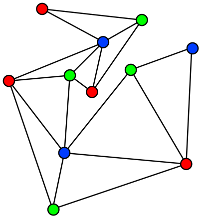
Some graphs of small treewidth:



Trees (tw $\leq$ 1)



Series-parallel (tw $\leq$ 2)



outerplanar (tw $\leq$ 2)

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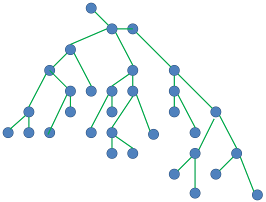

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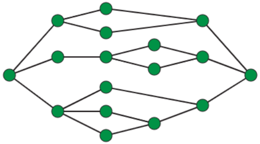

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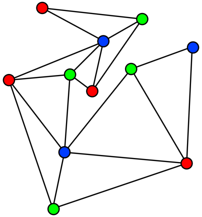
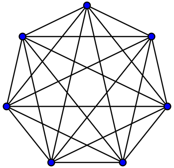
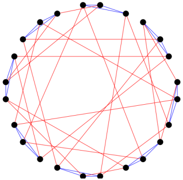


outerplanar (tw ≤ 2)

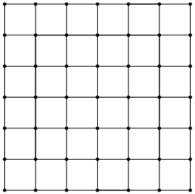
Some graphs of large treewidth:



Clique (tw $= n - 1$)



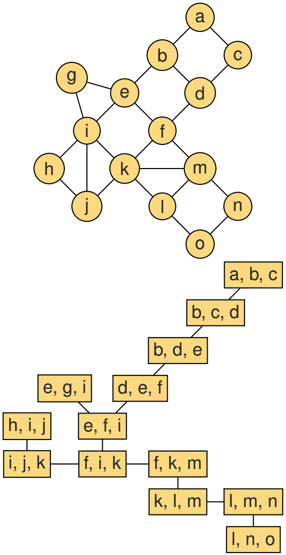
Expanders (tw $= \Theta(n)$)



$n \times m$-grid (tw $= \min(n, m)$)

Why treewidth?

- Algorithms for trees often generalize to algorithms for graphs of small treewidth



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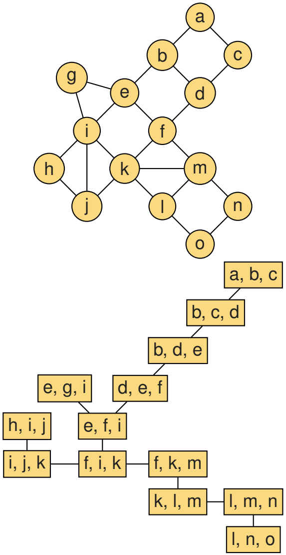

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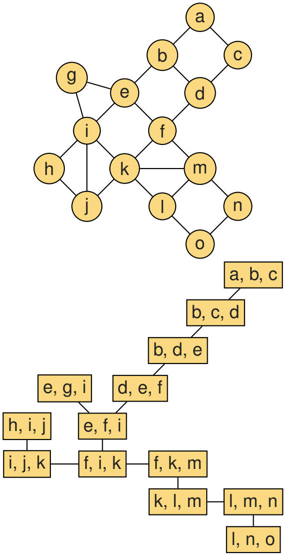

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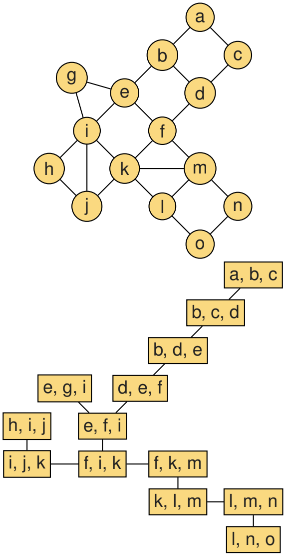

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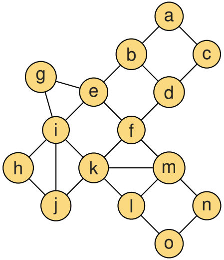
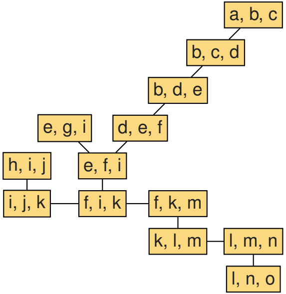
  - $2^{\mathcal{O}(k^3)} n$ time algorithm to compute an optimum-width tree decomposition [Bodlaender '96]
  - $2^{\mathcal{O}(k)} n$ time for 2-approximation [K. '21]
  - $n^{\mathcal{O}(1)}$ time for $\mathcal{O}(\sqrt{\log k})$-approximation [Feige,Hajiaghayi,Lee'08]





Dynamic treewidth

Question [Bodlaender '93, Dvořák, Kupec & Tůma '14, Alman, Mnich & Vassilevska Williams '20]

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Theorem (This work)

$2^{\mathcal{O}(k)} \log n$ amortized update time 9-approximate tree decomposition.

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There is data structure that

- is initialized with integer k and an edgeless n-vertex graph G
- supports edge insertions/deletions in amortized time $2^{\mathcal{O}(k)} \log n$ under the promise that $\mathrm{tw}(G) \leq k$
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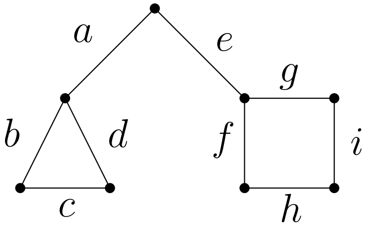
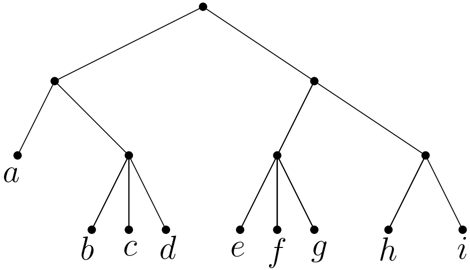
$\Rightarrow$ Dynamic Courcelle's theorem in $f(k) \cdot \log n$ amortized update time

The algorithm

Maintained decomposition





- Branch decomposition: Rooted tree whose leaves correspond to the edges of the graph

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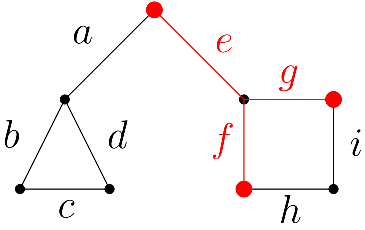

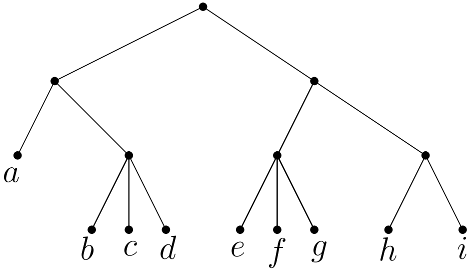

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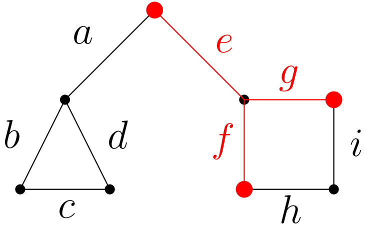

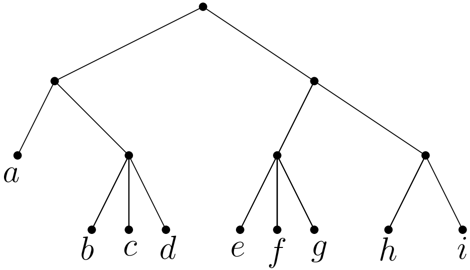

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- A set of edges $F \subseteq E$ is well-linked if it cannot be partitioned to (C_1, C_2) so that $|\partial(C_1)| < |\partial(F)|$ and $|\partial(C_2)| < |\partial(F)|$

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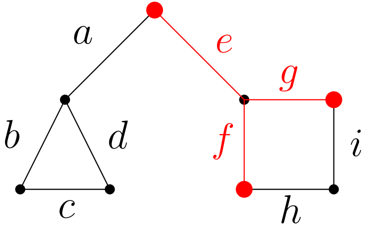

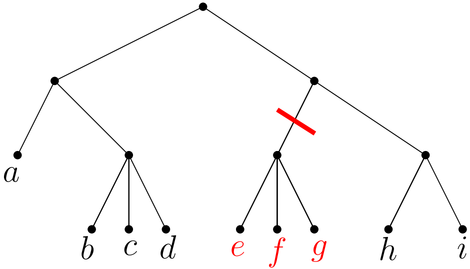

- Branch decomposition: Rooted tree whose leaves correspond to the edges of the graph
- Boundary $\partial(F)$ of a set of edges $F \subseteq E$: The vertices incident to edges from both F and $E \setminus F$.
- A set of edges $F \subseteq E$ is well-linked if it cannot be partitioned to (C_1, C_2) so that $|\partial(C_1)| < |\partial(F)|$ and $|\partial(C_2)| < |\partial(F)|$
- Want to maintain: Every edge set corresponding to a subtree is well-linked "downwards well-linkedness"

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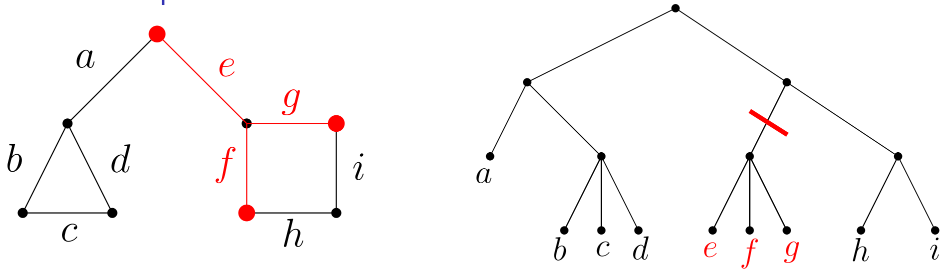

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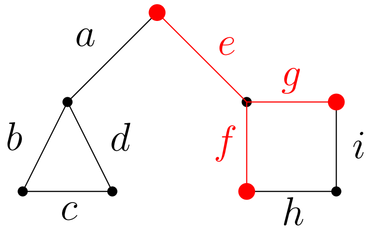

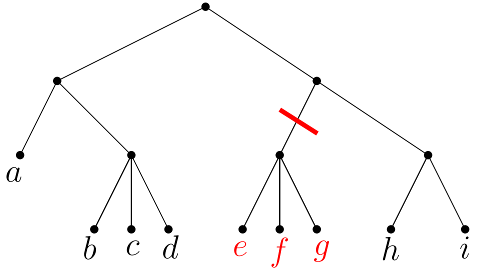

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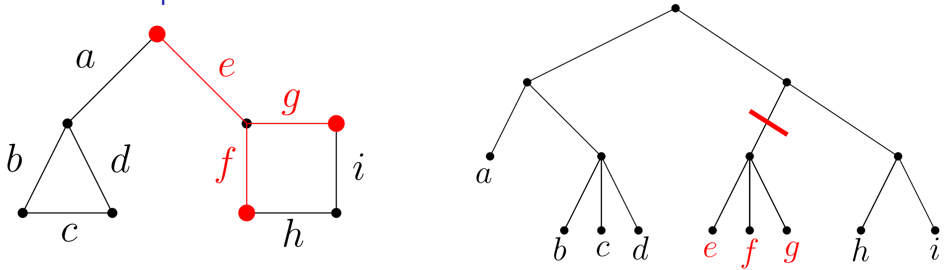

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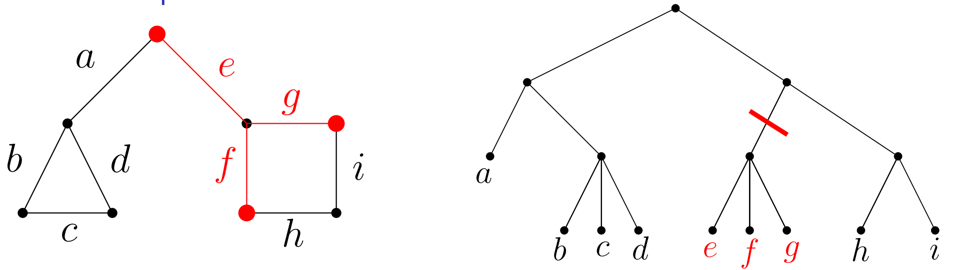

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Local rotations

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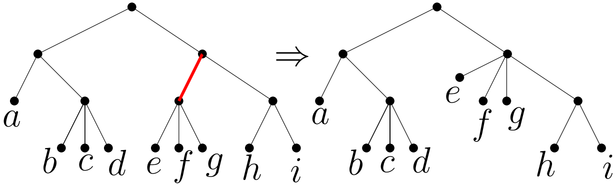

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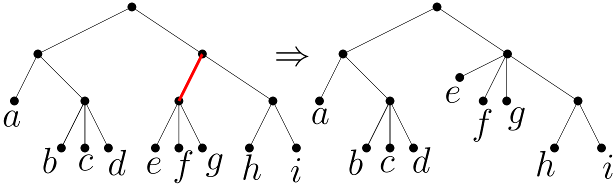

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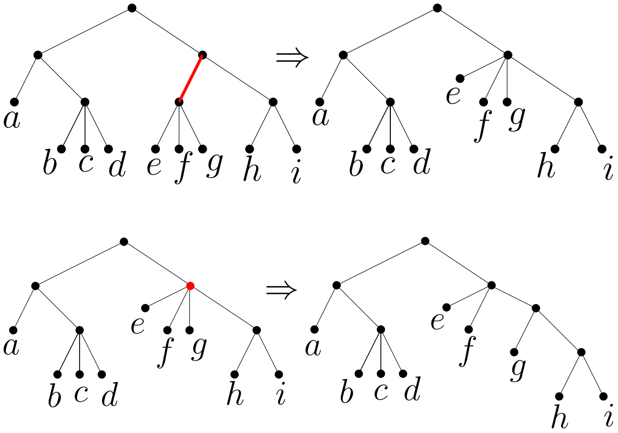

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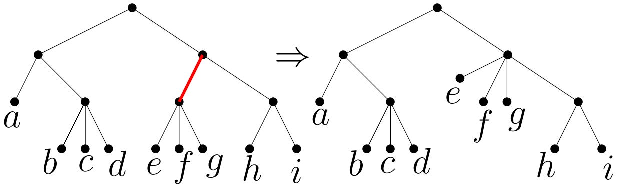

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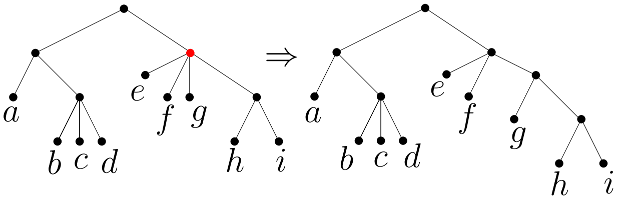

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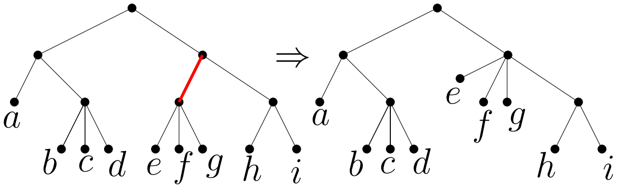

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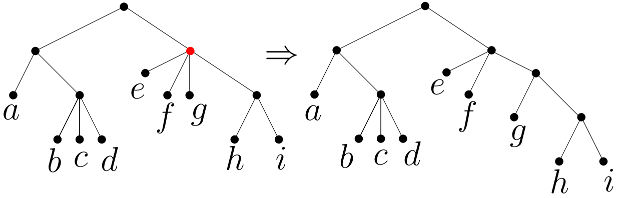
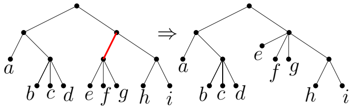
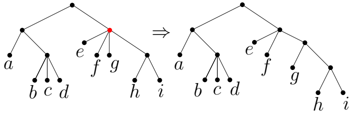


Approach

- **Idea**: Implement splay-tree-like rotations with the local rotations





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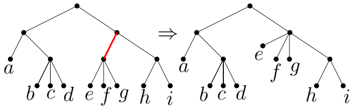

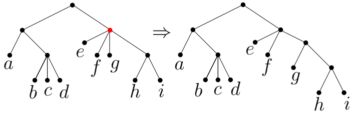

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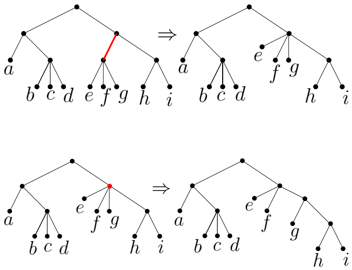

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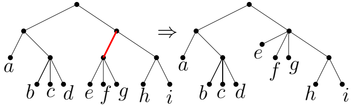

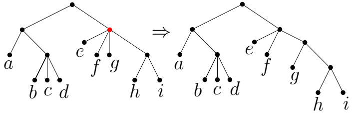

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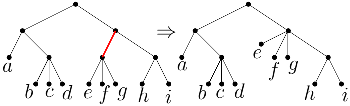

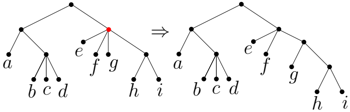

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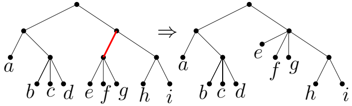
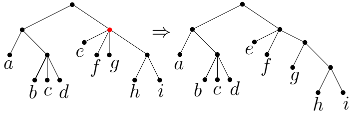
- **Last step:** Replace each node by a tree decomposition of width $\mathcal{O}(k)$





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